ENSC 3233
Chapter 8 Lecture 1

Hints for the period

The two most important equations in Chapter 6 are:

Darcy-Weisbach pipe friction head loss: $h_L = f \frac{l}{D}\frac{V^2}{2g}$

Minor (fittings) loss: $h_L = K_L \frac{V^2}{2g}$

Read Chapter 6
Not responsible for

Today we will:
Start Chapter 8, Viscous Flow in Pipes (or why we have a Reynolds number **Re**=$\rho VD/\mu$))

Review progress to date

We now have four basic tools of fluid mechanics.

- »Equations of State:
  - Hydrostatics
  - Viscosity
  - Gas Law/Compressibility
- »Continuity
- »Momentum
- »Energy

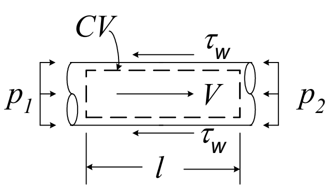
In other words you are about equal to an 1850 engineer.

What do we lack?

- »*The ability to predict friction (momentum transport)*

Friction

Free body inside a *horizontal, constant diameter* pipe



Momentum Eq.

$$\sum F_x = p_1 A - p_2 A - \tau_w \pi D l = \rho Q(V_2 - V_1)$$

With $V_1 = V_2$

$$\tau_w = \frac{A(p_1 - p_2)}{\pi D l} = \frac{D(p_1 - p_2)}{4l}$$

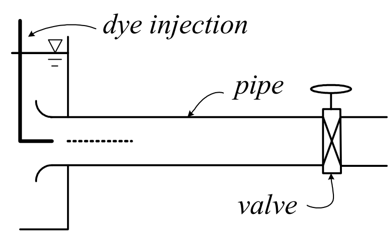
or

$$p_1 - p_2 = \frac{4l}{D}\tau_w$$

So to predict pressure drop we need the shear stress.

Reynolds' Work

Near the end of the nineteenth century Osborne Reynolds (1842-1912) was trying to figure out how to predict the pressure loss in pipes. One of his experiments was to measure flow in a simple pipe system.

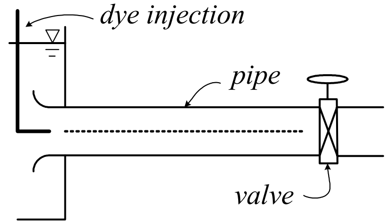


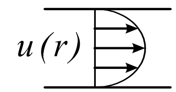
He ran the system for a lot of conditions, he changed the valve setting, the *inside diameter*, D, and even the liquid, i.e. μ. For each run he would record D, μ and Q. He could calculate the mean velocity, V in the pipe using continuity for incompressible fluids.

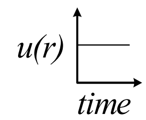
$$V = Q/A = Q/(\frac{\pi}{4}D^2)$$

As Hagen had before him, if he used a glass tube and injected dye he found something curious.

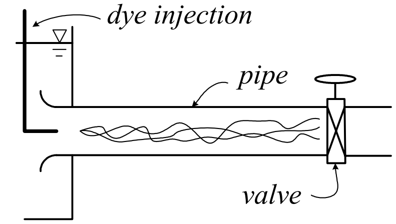
In *laminar (Re<2,300)* flow there is no mixing of the fluid, and instantaneous point velocities are the same as long term averages.

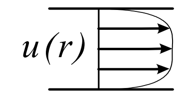




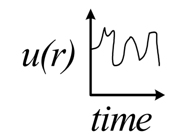


In *turbulent (Re>4,000)* flow the fluid is well mixed, and instantaneous point velocities are not equal to the long term averages.

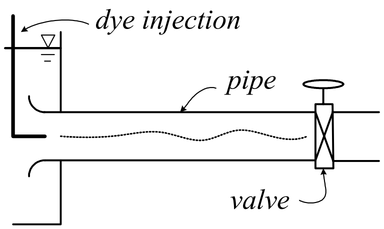




Long time average



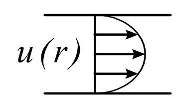
The *transition* range was hard to predict.



If you were to measure the long time point velocity, $u(r)$ you would find

laminar

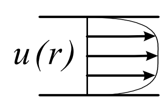
$$u(r) \propto \left(1 - \frac{r^2}{R^2}\right)$$



$$V = \frac{1}{2} u_{\max}$$

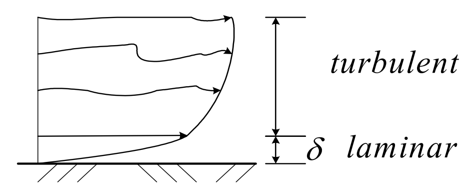
turbulent

$$u(r) \propto \left(1 - \frac{r}{R}\right)^{1/7}$$



$$V \approx u_{max}$$

If we could look very closely near the wall during turbulent flow, we could see this.



The turbulent flow has a laminar *viscous sublayer*. The laminar layer obeys Newton's law of viscosity,

$$\tau = \mu \frac{du}{dy}$$

The turbulent regions obey a new viscosity relationship

$$\tau = \eta \frac{d\bar{u}}{dy}$$

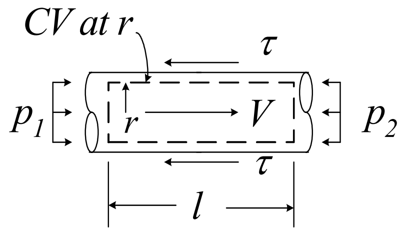
where η is the turbulent eddy viscosity. It is not constant, but is instead a function of the *Re*. In general $\eta >> \mu$.

Another way this is described is to call the turbulent shear stress the *Reynolds* Stress.

$$\tau = \mu \frac{du}{dy} - \rho \overline{u'} \overline{v'}$$

Note: Because turbulent flow is always three-dimensional the Reynolds *Stresses* must be analyzed in 3-D. (It gets pretty messy.)

Return to the pipe with *laminar* flow. Place a control volume at an arbitrary radius, *r*.



Flow area = πr^2

Shear surface area = $2\pi r l$

Momentum Eq. in x

$$\sum F_x = p_1 \pi r^2 - p_2 \pi r^2 - \tau 2\pi r l = \rho Q_r (V_2 - V_1)$$

Note again that $V_1 = V_2$

let $\Delta p = p_1 - p_2$

$$\frac{\Delta p}{l} = \frac{2\tau}{r}$$

or

$$\tau = \frac{\Delta p r}{2l}$$

$$\tau = \frac{\Delta p}{2l} r$$

Note at $r = 0;\ \tau = 0$

At $r = R,\ \tau = \tau_w$ *(wall shear stress)*

$$\tau_w = \frac{\Delta p}{2l} R$$

$$\frac{\Delta p}{2l} = \frac{\tau_w}{R}$$

$$\boxed{\tau = \frac{\tau_w r}{R} = \frac{2\tau_w r}{D}}$$

Return to

$$\tau = \frac{\Delta p}{2l} r$$

Newton's Viscosity Law in cylindrical coordinates

$$\tau = -\mu \frac{du}{dr}$$

Substitute into shear stress relationship and rearrange

$$du = -\left(\frac{\Delta p}{2\mu l}\right) r dr$$

Integrate

$$\int du = -\frac{\Delta p}{2\mu l} \int r dr$$

$$u = -\frac{\Delta p}{4\mu l} r^2 + C$$

Note at $r = R,\ u = 0$. Therefore

$$C = \frac{\Delta p}{4\mu l} R^2$$

$$u = \frac{\Delta p}{4\mu l}(R^2 - r^2)$$

Note from before:

$$\tau_w = \frac{\Delta p}{2l} R$$

solving for Δp

$$\Delta p = \frac{\tau_w 2l}{R}$$

substituting for Δp

$$u = \frac{\tau_w R}{2\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Maximum velocity occurs on the centerline, $r = 0$

$$u_{\max} = \frac{\tau_w R}{2\mu} = \frac{R^2 \Delta p}{4\mu l}$$

We are now up to about 1883.