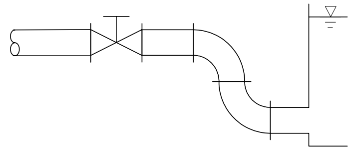
ENSC 3233
Chapter 6, Lecture 2

Homework
8.3, 8.5, 8.8, 8.12, 8.14, 8.18, 8.27, 8.31, 8.35

This lecture we will:
Poiseuille-Hagen Equation
Darcy-Weisbach Eq.
Moody Diagram

Hint for the period
In a complex system of pipes and fittings the total head loss is computed from the sum of the loss from each part.



$$h_f = \sum f \frac{l}{D}\frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

The energy equation becomes,

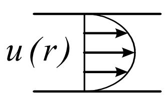
$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_f$$

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + \sum f \frac{l}{D}\frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

From before, for *laminar flow*

$$u = \frac{\tau_w R}{2\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

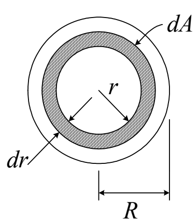
$$\tau_w = \frac{\Delta p}{2l} R$$



Integrate u over the pipe area to obtain Q

(Looking down pipe)

$q = u\,dA = u\,2\pi r dr$
(volume flow in donut)



$$Q = \int_{Area} u\,dA = \int_0^R q\,dr = \int_0^R u 2\pi r dr$$

(Note: $dA = 2\pi r dr$)

$$Q = \int_A u\,dA = \int_0^R \frac{\Delta p R^2}{4 l \mu}\left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r dr$$

Integration yields the ***Poiseuille-Hagen Equation***

$$\boxed{Q = \frac{\pi R^4 \Delta p}{8 \mu l} = \frac{\pi D^4 \Delta p}{128 \mu l}}$$ ***laminar flow & horizontal***

$$\boxed{Q = \frac{\pi D^4 h_f \gamma}{128 \mu l}}$$ **laminar general case**

Example: How does the average velocity relate to the maximum?

Average velocity,

$$V = Q/A = \frac{\pi R^4 \Delta p}{8 \mu l}\frac{1}{\pi R^2} = \frac{R^2 \Delta p}{8 \mu l}$$

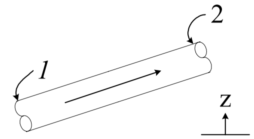
From last lecture

$$u_{\max} = \frac{\tau_w R}{2\mu} = \frac{R^2 \Delta p}{4 \mu l}$$

$$\frac{V}{u_{\max}} = \frac{\left(\dfrac{R^2 \Delta p}{8\mu l}\right)}{\left(\dfrac{R^2 \Delta p}{4\mu l}\right)} = \frac{1}{2}$$

(The mean velocity is ½ the maximum.)

Energy Equation on a pipe



(*Note point 2 is downstream from 1)*

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_f \text{ (no pumps \& turbines)}$$

$$h_f = \frac{V_1^2 - V_2^2}{2g} + \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)$$

$$\boxed{h_f = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)} \text{ (uniform pipe)}$$

Pipe Friction

What form does energy head losses, h_f have?

If we ran experiments with different pipes of various lengths, *but at the same V* we would find,

$$h_f \neq \phi(p)$$
$$h_f \propto l$$

If we ran experiments with the same kind of pipe, but with different diameters we would find

$$h_f \propto 1/D$$

Now if we ran pipes at various velocities we would find

$$h_f \propto V^n$$

$n = 1$ for all pipe with laminar flow
$n \approx 1.7$ smooth pipes with turbulent flow
$n = 2$ rough pipes at turbulent flow
Since we operate mostly rough pipes in high turbulent flow say

$$h_f \propto \frac{V^2}{2g}$$

Lets put it all together,

$$h_f \propto \frac{l}{D}\frac{V^2}{2g}$$

Introduce a proportionality parameter, the *Darcy friction factor f*

$$\boxed{h_f = f\frac{l}{D}\frac{V^2}{2g}} \text{ Pipe friction}$$

$$f = h_f / \frac{l}{D}\frac{V^2}{2g}$$

This is the *Darcy-Weisbach* relation for h_f. There are other relations around but they have mostly fallen out of use with the introduction of calculators.

Returning to the energy equation

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_f$$

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + f\frac{l}{D}\frac{V_2^2}{2g}$$

This is a form of the total energy equation where pipe friction is the only energy loss.

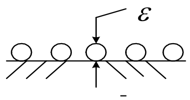
All we have to do is figure out *f*.

Lets do more experiments. Now what happens if we change the viscosity of the fluid, but not its density?

$f = \phi(1/\mu)$ for low V

$f \neq \phi(\mu)$ for high V

Likewise compare different pipes by their relative roughness, ε/D



$f \neq \phi(\varepsilon/D)$ for low V

$f = \phi(\varepsilon/D)$ for high V

TABLE 8.1
Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

Pipe	Equivalent Roughness, ε Feet	Equivalent Roughness, ε Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

If we changed ρ, keeping the other parameters constant, we would find

$f = \phi(1/\rho)$ for low V

$f \neq \phi(\rho)$ for high V

Using *dimensional analysis*. We know

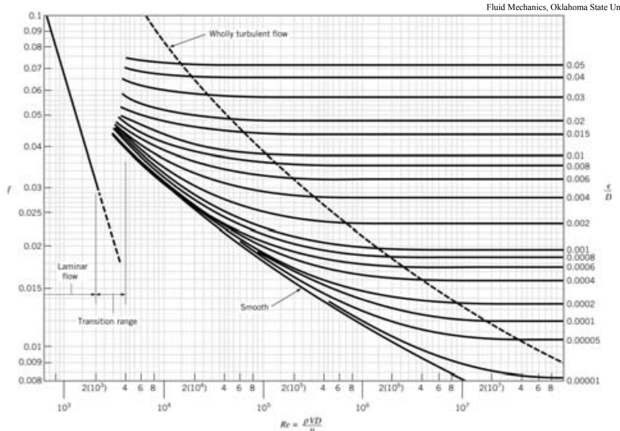
$f = \phi(V, D, \varepsilon, l, \mu, \rho)$

Note that f is already a dimensionless Π group.

$$f = h_f / \frac{l}{D}\frac{V^2}{2g} \doteq \frac{\text{L}}{\text{L}}$$

Buckingham Pi
$n = 5$ ($V, D, \varepsilon, \mu, \rho$)
j = 3 (M, L, T)
# Π = $n - j = 5 - 3 = 2$

Let D be the repeating variable. B*y inspection*
$\Pi_1 = \varepsilon/D$
$\Pi_2 =$ **Re** $= \rho VD/\mu$
So take all the data and plot it up as f vs Re with lines of constant ε/D



Example: Calculate the head loss for 100 m of 0.1m commercial steel pipe flowing water at 2 m/s.

$$\textbf{Re} = \frac{\rho VD}{\mu} = \frac{1000 \cdot 2 \cdot 0.1}{0.00112} = 1.8 \times 10^5$$

From Table 6.1
$\varepsilon = 0.046$ mm $= 4.6 \times 10^{-5}$ m
$\varepsilon/D = 4.6 \times 10^{-5}/0.1 = 0.00046$
From Moody, $f = 0.019$

$$h_f = \frac{flV^2}{D2g} = \frac{0.019 \cdot 100\ m}{0.1\ m}\frac{(2\ m/s)^2}{2 \cdot 9.81 \frac{m}{s^2}} = 3.9m$$