**ENSC 3233**
**Chapter 8, Lecture 3**

**Hint for period**
For laminar flow, **Re** < 2,100 the Darcy-Weisbach friction factor is given by,

$$\boxed{f = \frac{64}{\text{Re}}}$$

**This period we will**:
Examples of head loss calculations in pipes





**Compare Poiseuille and D-W equations**
For laminar flow, Poiseuille's law from before

$$Q = \frac{\pi D^4 \Delta p}{128 \mu l} \quad \textit{(horizontal \& laminar)}$$

*or,*

$$\Delta p = \frac{Q 128 \mu l}{\pi D^4}$$

Note Continuity: $Q = VA$

$$\Delta p = \frac{VA 128 \mu l}{\pi D^4} = \frac{V\pi \frac{D^2}{4} 128 \mu l}{\pi D^4}$$





$$\boxed{\Delta p = \frac{32 \mu l V}{D^2}} \quad \textit{(horizontal, laminar friction loss)}$$

*From Energy Equation*

$$h_L = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = \frac{\Delta p}{\rho g} \quad \textit{(horizontal)}$$

$$\boxed{h_L = \frac{\Delta p}{\rho g} = \frac{32 \mu l V}{\rho g D^2}}$$





By the D-W equation

$$h_L = f \frac{l}{D} \frac{V^2}{2g}$$

Set the two equations equal

$$\boxed{h_L = \frac{32 \mu l V}{\rho g D^2} = f \frac{l}{D} \frac{V^2}{2g}}$$

Solve for $f$

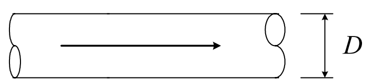
$$f = \frac{64}{\rho V D / \mu} = \frac{64}{\mathbf{Re}}$$

which is the equation for the "laminar line" on the Moody diagram





**Example**: A horizontal tube




$D$ = 1 cm = 0.01 m
$\mu$ = 0.0012 Ns/m$^2$
$\rho$ = 999 kg/m$^3$
What is the highest flow for laminar conditions?

Reynolds Number:

$$\text{Re} = \frac{\rho V D}{\mu} = 2{,}100 \text{ (maximum)}$$





Solve for $V$

$$V = \frac{\text{Re}\,\mu}{\rho D} = \frac{2{,}100 * 0.0012}{999 * 0.01} = 0.27 \text{m/s}$$

Apply Continuity

$$Q = VA = \frac{V \pi D^2}{4} = 2.2 * 10^{-5} \text{m}^3\text{/s}$$

**Example** What is the pressure drop in this pipe per meter of length ($l$ = 1 m)?

Energy Equation:

$$\frac{V_1^2}{2g}+\frac{p_1}{\rho g}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\rho g}+z_2+h_L$$

$$p_1-p_2=\rho g h_f=\rho g f\frac{l}{D}\frac{V^2}{2g}$$

Laminar $f$

$$f=\frac{64}{\text{Re}}=\frac{64}{2{,}100}=0.030 \quad (\textit{from before } \mathbf{Re}=2{,}100)$$





Substituting:

$$p_1-p_2=999\frac{kg}{m^3}*0.030*\frac{1m}{0.01m}\frac{(0.27m/s)^2}{2}$$

$$\Delta p = p_1 - p_2 = 110 \text{ N/m}^2$$

What is the wall shear stress?

$$\tau_w=\frac{\Delta p}{2l}R=\frac{110N/m^2}{2*1m}\frac{0.01\text{m}}{2}$$

$$\tau_w = 0.28 \text{ N/m}^2$$





**Example**

10 m of 3/8" steel Sch 80 pipe, water at 20 C, and $Q$ = 0.01 l/s. Find $h_L$.

$\rho$ = 1000 kg/m3 μ = 0.001 Ns/m2

I.D. = 1.07 cm $A$ = 9.06x10-5 m2

$\varepsilon$ = 0.0045 cm

*and*

$Q$ = 0.01 l/s = 10-5 m3/s (1,000 liters in 1 $m^3$)

Continuity:

$V = Q/A$ = 10-5/9.06-5 = 0.11 m/s





Reynolds Number:

**Re**=(1000kg/$m^3$ 0.11m/s 0.01m)/0.001Ns/$m^2$=1100

Note **Re** < 2100; we have laminar flow. Therefore,

$f$ = 64/**Re** = 64/1100 = 0.058

D-W Equation:

$$h_L=f\frac{l}{D}\frac{V^2}{2g}=0.058\frac{10m}{.0107m}\frac{(0.11m/s)^2}{2*9.81m/s^2}$$

$$h_f = 0.33 \text{ m}$$





**Example**

Same pipe and conditions as before, except $Q$ = 0.1 l/s = $10^{-4}$ $m^3$/s. Find $h_f$

Continuity:

$V = Q/A$ = 10-4/9.06-5 = 1.1 m/s

Reynolds Number:

**Re** = (999kg/$m^3$ 1.1m/s 0.01m)/0.001Ns/$m^2$=11,000

Note **Re** > 2100, we have turbulent flow, use Moody

Calculate ε/$D$:

ε/$D$ = 0.0046 cm/ 1.07 cm = 0.0044

Get $f$ from Moody diagram, (**Re** = 11,000)





$f$ = 0.036

D-W Eq.:

$$h_L=f\frac{l}{D}\frac{V^2}{2g}=0.036\frac{10m}{.0107m}\frac{(1.1m/s)^2}{2*9.81m/s^2}$$

$$h_L = 2.1 \text{ m}$$

***Find $h_L$ with Q, and D known***

1. Calculate *V,*

$$V = Q/(\pi D^2/4)$$

2. Calculate **Re**

$$\mathbf{Re} = \rho VD/\mu = VD/\nu$$

3a. If **Re** < 2,300 use $f = 64/Re$

3b. If **Re** > 4,000 find $f$ on Moody diagram with $\varepsilon/D$

3c. If 2,100 < **Re** < 4,000 estimate $f$ from Moody

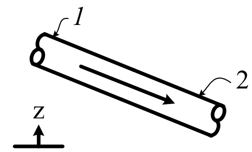
4. Use D-W formula for $h_L$

$$h_L = f\frac{l}{D}\frac{V^2}{2g}$$





**Example**: A simple pipe system. Find $Q$.




3/8" steel Sch 80 pipe

$L = 100$ m

Water at 20 C

$z_1 = 10$ m

$p_1 = 10{,}000$ N/m$^2$

$z_2 = 2$ m

$p_2 = 11{,}000$ N/m$^2$





Look up parameters

$\rho = 1{,}000$ kg/m$^3$

$\mu = 0.001$ Ns/m$^2$

I.D. = 1.07 cm = 0.0107 m (Handout)

$A = 0.906$ cm$^2$ = 9.06x10$^{-5}$ m$^2$

$\varepsilon$=0.0046 cm (Table 6.1)

Energy Eq. (head form)

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_L$$

Solve for $h_L$





(note $V_1 = V_2$)

$$h_L = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)$$

$$h_L = \frac{(10{,}000 - 11{,}000)\text{Ns/m}^2}{1000\,\text{kg/m}^3 9.81\text{m/s}^2} + (10 - 2)\text{m}$$

$h_L = 7.9$ m

D-W Equation:

$$h_L = f\frac{l}{D}\frac{V^2}{2g}$$

Solve for $V$:





$$V = \sqrt{\frac{h_L D 2g}{fl}}$$

Looks like trouble, we have one equation, but two unknowns, $V$ and $f$.

Lets guess a $f$ value. First calculate the relative roughness,

$\varepsilon/D$ = 0.0046 cm/ 1.07 cm = 0.0044

Use Moody chart to estimate $f$ based on $\varepsilon/D$

Assume $f = 0.028$

(this assumes complete turbulence, or large **Re**)





$$V = \sqrt{\frac{h_L D 2g}{fl}}$$

$$V = \sqrt{\frac{7.9\text{m} * 0.0107\text{m} * 2 * 9.81\text{m/s}^2}{0.028 * 100\text{m}}}$$

$V = 0.77$ m/s

*Check* **Re** *on Moody diagram*

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{1000 * 0.77 * 0.0107}{0.001}$$

**Re** = 7,700

Check Moody chart, $f$ doesn't match

Try again assume $f = 0.036$

$V = 0.68$ m/s

**Re** = 6773 **OK**

$Q = VA = 0.68 * 9.06\text{x}10^{-5}$

$Q = 6.0\text{x}10^{-5}\ \text{m}^3/\text{s}$





**Calculate *Q or V* with known $h_f$ and *D***

- Calculate $\varepsilon/D$
- Assume a $f$ based on $\varepsilon/D$ using Moody Diagram
- Calculate $V$ by

$$V = \sqrt{\frac{h_L D 2g}{f l}}$$

- Calculate **Re** and check $f$ on Moody
- Repeat if $f$ does not match

And don't forget for *laminar flow,* **Re** < *2,100*

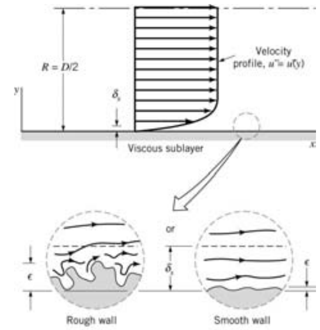
$f = 64/\textbf{Re}$





**One last concept from pipe friction.** Why does a smooth pipe behave differently from a rough one?

In a "smooth pipe" the surface roughness is always submerged by the viscous sublayer while in a "rough" pipe most of the roughness protrudes into the turbulent flow.








The thickness of the viscous sublayer is about,

$$\delta_s \approx 5\frac{\nu}{u*}$$

Where $u*$ is the friction (or shear) velocity

$$u* = \sqrt{\frac{\tau_w}{\rho}}$$

From the first lecture

$$\tau_w = \frac{D\Delta p}{4l}$$

Solving for $\delta_s$

$$\delta_s \approx 10\nu\sqrt{\frac{l\rho}{D\Delta p}} \approx 10\mu\sqrt{\frac{l}{\rho D\Delta p}}$$