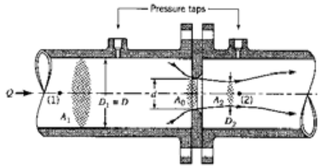
ENSC 3233
Chapter 8, Lecture 5

Homework:
8.53, 8.60, 8.68, 8.103, 8.110, 8.112
8.62 (use 4” sch 40 steel pipe)
8.71 (use ½” sch 80 steel pipe)
8.82 (use sch 40 steel pipe sizes)
This period
Flow metering devices

Pipe Flow Measurement

An accurate, low cost method of flow measurement in pipes is to restrict the flow area with an orifice plate, nozzle or Venturi.



An Orifice

For ideal flow there would be no energy losses and we could apply the Bernoulli equation between the pipe and the flow restriction.

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2$$

Continuity gives the ideal flow,

$$V_1=V_2\frac{A_2}{A_1}=V_2\frac{\pi D_2^2/4}{\pi D_1^2/4}=V_2\frac{D_2^2}{D_1^2}=V_2\beta^2$$

where $\beta = D_2/D_1 = d_{throat}/D_{pipe}$

Substituting into the Bernoulli eq.

$$V_2=\sqrt{\frac{2\left[\frac{p_1-p_2}{\rho}+g(z_1-z_2)\right]}{(1-\beta^4)}}$$

Continuity also states

$$Q_{ideal} = V_2 A_2$$

$$Q_{ideal}=A_2\sqrt{\frac{2\left[\frac{p_1-p_2}{\rho}+g(z_1-z_2)\right]}{(1-\beta^4)}}$$

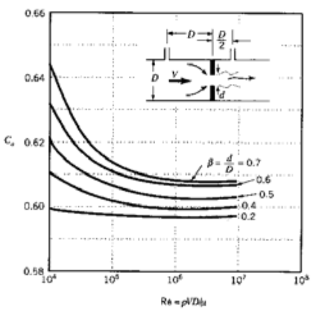
For real meters,

$$Q_{ideal}=C_d A_2\sqrt{\frac{2\left[\frac{p_1-p_2}{\rho}+g(z_1-z_2)\right]}{(1-\beta^4)}}$$

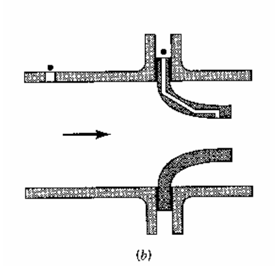
Where A_2 is the constriction area of the throat and C_d is a calibration coefficient that is a function of **Re** and β. (Note A_o , A_n, A_T, C_o, C_n, C_v used in the book) For a horizontal placement, $z_1 = z_2$

$$Q=C_d A_2\sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}$$

Orifice,
C_d is determined with Figure 8.41



Nozzle,



C_d is a function of Re and $\beta = d/D$ and is plotted on Figure 8.43

$0.94 < C_d < 0.995$

Venturi

■ FIGURE 8.44 Typical Venturi meter construction.

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C_d is determined with Figure 8.45 (lame example) of actual curves.

$C_d \cong 0.98$

For Venturi's the **Re** in the plot is usually calculated at the throat, V_2

Example: A horizontal nozzle with water. Find Q.

p_1 = 144 psi = 1 lb/ft^2

p_2 = 72 psi = 1/2 lb/ft^2

D_1 = 12" = 1 ft

D_2 = 9" = 0.75 ft

$\rho = \gamma/g$=62.4 lb/ft^3/32.2 ft/s^2=1.94 slug

Find Q

$$Q = C_d A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$$

$\beta = 0.75ft/1.0\ ft = 0.75$

$A_2 = \pi(0.75ft)^2 / 4 = 0.442\ ft^2$

$$Q = C_d\, 0.442 \sqrt{\frac{2(1-0.5)}{1.94(1-0.75^4)}}$$

$Q = 0.534\ C_d$

From Figure 8.43 guess $\mathbf{Re_d} = 10^5$

$C_d = 0.98$;

$Q = 0.534*0.98 = 0.523\ ft^3/s$

Check **Re** *at* A_2

$V = Q/A_2 = 0.523/(\pi 0.75^2/4)=1.19\ ft/s$

Re = $(1.94 * 1.19 * 1) / 2.34*10^{-5}$

Re = $9*10^4$ *Try again, with this value of* **Re**

$C_d=0.97$; $\underline{Q=0.518\ ft^3/s}$;

Re$=5*10^4$ *checks*

$\boxed{Q = 0.518\ ft^3/s = 232\ gal/min}$

Flow Meters

Know D_1, D_2,& p_1-p_2; Find Q

1. Calculate $\beta = D_2/D_1$
2. Calculate value for *Constant* =

Horizontal

$$A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$$

Vertical

$$A_2 \sqrt{\frac{2\left[\frac{p_1 - p_2}{\rho} + g(z_1 - z_2)\right]}{(1-\beta^4)}}$$

3. Estimate value of discharge coefficient C from graph.
(Assume **Re** = 10^5) for first guess and make certain to use the appropriate area (usually A_{pipe} for orifices and nozzles, and A_{throat} for venturies) for the curve
4. Calculate discharge

$$Q_{(guess)} = C_{d(guess)} * Constant$$

5. Calculate **Re** and check C_d.
6. Repeat with new C_d as needed.

Example: A Vertical Flow Meter
Know D_1, D_2, & Q, Find $p_1 - p_2$

From before

$$Q_{ideal} = A_2\sqrt{\frac{2\left[\frac{p_1 - p_2}{\rho} + g(z_1 - z_2)\right]}{(1-\beta^4)}}$$

Note

$$Q_{ideal} = Q/C_d$$

Solve for $p_1 - p_2$

$$p_1 - p_2 = \rho\left[\frac{(1-\beta^4)}{2}\left(\frac{Q}{C_d A_2}\right)^2 - g(z_1 - z_2)\right]$$

Flow Meters

Know D_1, D_2, & Q, Find $p_1 - p_2$
1. Calculate $\beta = D_2 / D_1$
2. Calculate **Re** get C_d from graph.

3. Calculate either,
Horizontal meter

$$p_1 - p_2 = \frac{\rho(1-\beta^4)}{2}\left(\frac{Q}{C_d A_2}\right)^2$$

Vertical meter

$$p_1 - p_2 = \rho\left[\frac{(1-\beta^4)}{2}\left(\frac{Q}{C_d A_2}\right)^2 - g(z_1 - z_2)\right]$$